WEAKLY DISTURBED STEADY TRANSONIC FLOWS OF A VIBRATIONALLY RELAXING GAS

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The formation of shock waves in transonic flows is a problem which is far from being resolved. Conventional gas dynamics contains numerous examples of the construction of transonic flows free of discontinuities and flows in which discontinuities occur in the transition through the speed of sound (see [1], for example). Experiments have confirmed the existence of both continuous [2] transonic flows and transonic flows containing discontinuities [3]. The exceptional nature of nonshock flow in a local supersonic region was shown in [4], which suggested to one author that continuous stagnation of a transonic flow is not possible [5]. In the opinion Kuo and Sirs [6], such a flow is either unstable or contains shock waves or both. One possible approach to solving this problem is the study of transonic flows for their stability in relation to steady and nonsteady disturbances that might develop in the flow due to irregularities on the walls bounding the flow or the arrival of weak nonsteady waves at the sonic line. Such an investigation is complex in the general case, with approximate methods usually being used to obtain a solution.

The stability of a transonic flow relative to small nonsteady disturbances in a small neighborhood of the sonic line was examined in [7, 8] on the basis of a quasi-unidimensional approximation. An attempt to perform an analysis in general form was made in [9], where results agreeing with the data in [7] were obtained after certain simplifying assumptions were made. Kuz'min [10] analyzed the stability of a transonic flow on the basis of linearization of the Linn-Reissner-Tsien equation. It was found that transonic flow is stable in relation to a small change in the shape of the walls of the nozzle and the conditions at the inlet if the acceleration of the specified flow is everywhere positive.

The examples of steady continuous transonic flows that have been constructed exist only near certain types of solid boundaries, thus giving rise to the view that such flows are the exception. A study of the behavior of steady perturbations of such flows due to irregularities in the walls bounding the flow will make it possible to study the development of the features of the flow.

Aspects of the formation of shock waves and the conditions for nonshock flow in transonic flows were examined in [11]. It was shown that shock waves form if the slope of the walls of the channel near the mouth of the nozzle is too gentle. Discontinuities may form at a point on the sonic line or downstream, in the supersonic region. Numerical calculations of plane transonic flow with a local supersonic zone have shown that a shock wave is formed inside the region of supersonic flow [12].

Below, we examine the behavior of steady and nonsteady perturbations of steady transonic flows of a vibrationally relaxing gas.

Use of the methods in [8] makes it possible to reduce the problem of determining the stability of a transonic flow against small nonsteady distortions to the study of a certain nonlinear partial differential equation. Analysis of the solutions of this equation for specific states of the medium shows that flow with a transition from the supersonic regime to the subsonic regime is stable relative to small distortions if the gas is in the equilibrium state or if it is nonequilibrium with unexcited vibrational degrees of freedom. In this case, transonic flow will also be stable in the course of the transition from the subsonic to the supersonic regime.

In our study of the behavior of steady distortions, we examined the supersonic region of transonic flow. We derive the equations of the transonic approximation as in traditional

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gas dynamics. We do not suppose that the parameters in the perturbation region are necessarily small relative to the deviations of their values from the values at the speed of sound (transonic approximation). We employed the shortwave approximation [13] to analyze the behavior of the perturbations. It was found that the latter decay in an accelerating flow and grow in decelerating flow. The relaxation process connected with the excitation of vibrational degrees of freedom of the molecules leads to additional decay of the perturbations.

Thus, a continuously stagnating transonic flow may exist in a stream of vibrationally relaxing gas. As a result, continuous transonic flow may also occur about a body in the stream. Such flow may be accompanied by the formation of local supersonic zones.

1. The system of equations describing plane or axisymmetric steady flow of a gas with allowance for vibrational relaxation has the form

$$(\rho u)_x + (\rho v)_y + \frac{\nu\rho v}{y} = 0,\ \rho(uu_x + vu_y) + p_x = 0,\ \rho(uv_x + vv_y) + p_y = 0,$$
$$up_x + vp_y - a^2(u\rho_x + v\rho_y) = -\rho(\gamma-1)(ue_{kx} + ve_{ky}),$$
$$ue_{kx} + ve_{ky} = \omega\left(e_k^* - e_k\right) = F,$$

where ρ is density; p is pressure; u and v are the components of velocity along the x and y axes, respectively; e_k and e_k^* is the energy associated with the vibrational degrees of freedom of the molecules of the gas and its equilibrium value; a is the stagnated speed of sound; ω is the inverse relaxation time; γ is the adiabatic exponent; ν = 0 and 1 for plane and axisymmetric flows; and the subscripts x and y denote differentiation with respect to the corresponding coordinate.

We take the following relations [14] for e_k^* and ω

$$\omega = k_1 p\exp\left(-k_2T^{-1/3}\right),\quad e_k^* = R\theta_k/(\exp(\theta_k/T) - 1).$$

Here, R is the gas constant; T is the translational temperature; θ_k is the characteristic vibrational temperature; and k_1 and k_2 are vibrational constants dependent on the properties of the gas [14].

The linear theory of steady-state flow of a vibrationally relaxing gas [15] gives

$$u = u_0(1 - \delta\Phi_{xx}),\quad v = -u_0\delta\Phi_{xy},\quad p = \gamma p_0\left(\frac{1}{\gamma} - \delta M_0^2\Phi_{xx}\right),$$
$$\rho = \rho_0(1 + \delta\Phi_{xx} + \delta\Phi_{yy}),\quad e_k = a_0^2\left(\bar e_{k0} + \frac{1}{\gamma-1}\left(\delta\left(1 - M_0^2\right)\Phi_{xx} + \delta\Phi_{yy}\right)\right), \tag{1.2}$$

where $\Phi_{xx} = f''$; $\Phi_{xy} = -\sqrt{M_0^2 - 1}\,(f'' + \Lambda f')$; $\Phi_{yy} = \left(M_0^2 - 1\right)(f'' + 2\Lambda f' + \Lambda^2 f)$; $f'' = \left(\Upsilon_x - \Lambda\int\limits_0^x \Upsilon_\eta \exp(\Lambda(\eta - x))\,d\eta\right)/\sqrt{M_0^2 - 1}$; $\delta \ll 1$; M is the Mach number; Λ is (1.2) a parameter characterizing the relaxation process; and Y is a function determining the initial profile of the perturbation. It is evident that at $M_0^2 \to 1$, $\Phi_{xx} \to \infty$, $\Phi_{yy} \to 0$ Φ_{xy} is finite. Thus, the flow parameters u, p, and ρ increase to infinity and e_k and v approach constants. We will examine a transonic flow which is close to a constant equilibrium flow with sonic velocity along the x axis (a zero subscript will be used to denote the parameters of this flow). The above-obtained asymptotes (1.2) substantiate the use of the following flow-parameter expansion in the study of transonic flow:

$$u = a_0(1 + \tau u_1),\ v = -a_0\tau^{3/2}v_1,\ \ p = \gamma p_0\left(\frac{1}{\gamma} + \tau p_1\right),\ \rho = \rho_0(1 + \tau\rho_1),\ e_k = a_0^2(e_{k0} + \tau^2 e_{k2}). \tag{1.3}$$

Here, we change over to new coordinates: $\bar x = x/L,\ \bar y = y\sqrt{\tau}/L$. (L is the characteristic length). We also introduce the quantities

$$\bar\omega = \omega L/u_0,\ \bar e_k^* = e_k^*/a_0^2.$$

Using (1.3), we obtain

$$u^2 - a^2 = a_0^2\tau(2u_1 - \gamma p_1 + \rho_1),\ \ \sqrt{M^2 - 1} = \sqrt{\tau}\sqrt{2u_1 - \gamma p_1 + \rho_1}.$$

We reduce the equations of system (1.1) to their characteristic form [13]. Making use of the notation below

$$\frac{d_\pm}{dt}=\frac{\partial}{\partial x}+\frac{uv\pm a^2\sqrt{M^2-1}}{u^2-a^2}\frac{\partial}{\partial y},\quad \frac{d_0}{dx}=\frac{\partial}{\partial x}+\frac{v}{u}\frac{\partial}{\partial y},$$

we have

$$\begin{gathered}
v\frac{d_\pm u}{dx}-u\frac{d_\pm v}{dx}\mp\frac{\sqrt{M^2-1}}{\rho}\frac{d_\pm p}{dx}=\left[\frac{(\gamma-1)F}{u^2-a^2}+\frac{a^2 v\nu}{y(u^2-a^2)}\right]\times\\
\times\left(\pm u\sqrt{M^2-1}+v\right),\\
\frac{d_\pm y}{dx}=\frac{uv\pm a^2\sqrt{M^2-1}}{u^2-a^2},\quad \rho\frac{d_0}{dx}\left(\frac{u^2+v^2}{2}\right)+\frac{d_0 p}{dx}=0,\\
\frac{d_0 p}{dx}-a^2\frac{d_0\rho}{dx}=-\frac{\rho(\gamma-1)F}{u},\quad \frac{d_0 e_k}{dx}=\frac{F}{u},\quad \frac{d_0 y}{dx}=\frac{v}{u}.
\end{gathered}\tag{1.4}$$

The first equation of system (1.4) is written along the acoustic characteristic. The equation of this characteristic is reduced to the equation of the second characteristic (the characteristic pertains to the first family when the top sign is chosen and to the second family when the bottom sign is chosen). The following three equations are written along the streamline. The equation of the streamline is the last equation in system (1.4).

Retaining the principal (in terms of τ) terms, after we insert expansion (1.3) into system (1.4) we obtain the equations of the transonic approximation (the terms that remain have the order $\tau^{3/2}$ in the first equation and the order τ in the second equation; it is assumed that $\bar{\omega}_0$ has the order τ; the bars above x and y are omitted):

$$\begin{gathered}
-\frac{d_\pm v_1}{dx}\mp\sqrt{2u_1-\gamma p_1+\rho_1}\frac{d_\pm p_1}{dx}=\pm\frac{1}{\sqrt{2u_1-\gamma p_1+\rho_1}}\times\left[(\gamma-1)\frac{\bar{\omega}_0}{\tau}\bar{e}^*_{k1}+\frac{v_1\nu}{y}\right],\\
\frac{d_\pm y}{dx}=\pm\frac{1}{\sqrt{2u_1-\gamma p_1+\rho_1}},\quad \frac{d_0 u_1}{dx}+\frac{d_0 p_1}{dx}=0,\quad \frac{d_0 p_1}{dx}-\frac{d_0\rho_1}{dx}=0,\quad \frac{d_0 y}{dx}=0
\end{gathered}\tag{1.5}$$

$\left(\bar{e}^*_{k1}=\gamma\bar{e}^{*2}_{k0}\exp\frac{\theta_k}{T_0}(\gamma p_1-\rho_1)\right)$. We omit the equations of system (1.4) which determines e_k and would give principal terms of the order τ^2. The first two equations of system (1.5) have meaning only when $2u_1 - \gamma p_1 + \rho_1 > 0$ (the region of supersonic flow).

Let there be a steady-state distortion in the flow due to irregularities on the walls bounding the flow. The parameters of the determined flow can be represented by the sums

$$u_1+u_{11},\ v_1+v_{11},\ p_1+p_{11},\ \rho_1+\rho_{11},$$

where the first terms correspond to the known transonic flow and the second terms correspond to the steady-state distortion.

The parameters of the undisturbed flow can be determined in the form of a series in the space coordinates x and y. Having connected the origin of the coordinate system with the sonic line and examining flow in a small neighborhood of this line, we can restrict ourselves to allowance for the leading terms of the series. It is convenient to represent system (1.5) in the following form. Choosing the top sign in the first equation of the system, we obtain

$$-\frac{dv_1}{dx}-\sqrt{2u_1-\gamma p_1+\rho_1}\frac{dp_1}{dx}=\frac{1}{\sqrt{2u_1-\gamma p_1+\rho_1}}\left[(\gamma-1)\frac{\bar{\omega}_0}{\tau}\bar{e}^*_{k1}+\frac{v_1\nu}{y}\right].\tag{1.6}$$

Adding (1.6) and the first equation of system (1.5) after choosing the bottom sign in the latter, we have

$$v_{1x}+p_{1y}=0.\tag{1.7}$$

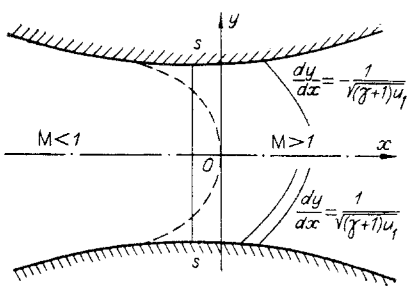

Fig. 1

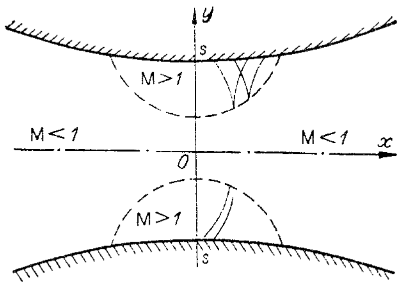

Fig. 2

then differentiate Eq. (1.6) with respect to x and use Eqs. (1.5) and (1.7) to obtain a nonlinear partial differential equation in u_1:

$$((\gamma+1)\,u_1u_{1x})_x - u_{1yy} - \frac{\nu}{y}\,u_{1y} + Eu_{1x} = 0$$
$$\left(E = \frac{1}{\tau}\,\gamma\,(\gamma-1)^2\,\bar{\omega}_0 \bar{e}_{k0}^{*2} \exp\frac{\theta_k}{T_0}\right). \qquad (1.8)$$

In conventional gas dynamics (in the absence of relaxation $\bar{\omega}_0 \to 0$) (1.8) we obtain the equation of the Karman–Guderlei approximation [16]:

$$-(\gamma+1)\,\Phi_x^0\Phi_{xx}^0 + \Phi_{yy}^0 + \frac{\nu}{y}\,\Phi_y^0 = 0 \qquad (1.9)$$

($\Phi_x{}^0 = u_1$). Equations (1.8) and (1.9) are of the mixed type: hyperbolic at $(\gamma+1)u_1 > 0$, $(\gamma+1)\,\Phi_x{}^0 > 0$ and elliptic at $(\gamma+1)u_1 < 0$, $(\gamma+1)\,\Phi_x{}^0 < 0$. Two types of steady transonic flows named after their discoverer – Maier [16] and Taylor [17] – were constructed for (1.9). The parameters of the flows are represented in the form a of series in the space coordinates x and y (see Figs. 1 and 2 for the introduction of the coordinates). We construct similar solutions for Eq. (1.8).

For a flow of the Maier type, we assign a change in the longitudinal velocity $\bar{u}$ which is linear with respect to x (Maier hypothesis). In addition, for physical reasons we assume that the flow is symmetric relative to the x axis (the expansion contains the terms y^2, $y^4 \ldots$). Instead of the components of velocity u_1, v_1, it is convenient to introduce the flow potential Φ such that $u_1 = \Phi_x$, $v_1 = \Phi_y$. The coefficients of the potential expansion in x and y can be obtained by substitution of the expansion into Eq. (1.9):

$$\Phi_m = \frac{\alpha}{2}\,x^2 + \frac{\alpha\,(E+\alpha\,(\gamma+1))}{2\,(\nu+1)}\,xy^2 + \frac{\alpha\,(E+\alpha\,(\gamma+1))^2}{8\,(\nu+1)\,(\nu+3)}\,y^4. \qquad (1.10)$$

Flow with such a potential is an accelerating transonic flow at $\alpha > 0$.

We can use the potential to determine the streamlines and, thus, the boundaries of the flow. We find from the boundary conditions that

$$\alpha = \frac{1}{2\,(\nu+1)}\sqrt{E^2 + \frac{4\,(\gamma+1)\,(\nu+1)}{R_s y_s}},$$

where y_s is the ordinate of the minimum cross section of the nozzle; and R_s is the curvature of the nozzle in the minimum section.

The sonic line is determined by the equality

$$u_1 = 0. \qquad (1.11)$$

Using (1.11) and (1.10), we can obtain the equation of the sonic line

$$x = -\frac{1}{4\,(\nu+1)}\left(E + \sqrt{E^2 + \frac{4\,(\gamma+1)\,(\nu+1)}{R_s y_s}}\right) y^2.$$

The difference from conventional gas dynamics lies in the absence of the parameter $E > 0$. The sonic line in a relaxing gas is located farther downstream (except for the point $x = y = 0$ and is, therefore, more curved.

Taylor flow is also characterized by symmetry relative to the x axis. Insertion of an expansion of general form into Eq. (1.9) yields

$$\Phi_t = -\eta x - \frac{E}{2(\gamma+1)}x^2 - \frac{\beta}{3}x^3 + \frac{(\gamma+1)\eta\beta}{\nu+1}xy^2$$
$$\left(\beta = 2\frac{\gamma+1}{\nu+1}\eta y_s R_s > 0, \quad \eta = 1 - \frac{u}{a} > 0, \ \eta \ll 1\right).$$

The sonic line is determined from Eq. (1.11) and has the form

$$\eta + \frac{E}{\gamma+1}x + \beta x^2 = \frac{(\gamma+1)\eta\beta}{\nu+1}y^2.$$

In conventional gas dynamics ($\omega_0 \to 0$, $E \to 0$), the sonic line bounds two regions of supersonic flow around the walls of the nozzle. These regions are symmetric relative to the y axis (the flow field is also symmetric relative to the x axis). Relaxation disturbs the symmetry which exists relative to the y axis and displaces the supersonic region upstream.

For the quantities with the subscripts 11, corresponding to the perturbed state, we find from system (1.5) that

$$-\frac{dv_{11}}{dy} \pm \frac{2}{3}\sqrt{\gamma+1}\frac{d}{dy}\left[(u_{11}+u_1)^{3/2} - u_1^{3/2}\right] = \pm\left[\frac{v_{11}\nu}{y} - Eu_{11}\right],$$
$$\frac{d_\pm y}{dx} = \pm\frac{1}{\sqrt{(\gamma+1)(u_1+u_{11})}}, \quad u_{11} = -p_{11}, \ p_{11} = \rho_{11}. \tag{1.12}$$

The system which has been obtained shows that the given disturbance can be represented as a set of waves (acoustic, entropic-vortical, relaxation) propagating along the characteristics of the corresponding families, If the characteristic length of a flow-disturbing irregularity on the wall L is sufficiently small, then the perturbation waves created by it will be short (from the leading to the trailing edges) and we will be able to ignore the mutual effect of perturbations propagating along the characteristics of different families. Let us examine an acoustic wave of the first family, determined by the first equation of system (1.12), with the selection of the top sign in this equations:

$$-\frac{dv_{11}}{dy} + \frac{2}{3}\sqrt{\gamma+1}\frac{d}{dy}\left[(u_{11}+u_1)^{3/2} - u_1^{3/2}\right] = \frac{v_{11}\nu}{y} - Eu_{11}.$$

The remaining perturbations will be considered zero perturbations:

$$\frac{dv_{11}}{dy} + \frac{2}{3}\sqrt{\gamma+1}\frac{d}{dy}\left[(u_{11}+u_1)^{3/2} - u_1^{3/2}\right] = 0.$$

This assumption is valid for Maier flows if the disturbances are due to only to small irregularities in the solid boundaries (walls) in the supersonic region of the flow. After having developed, such perturbations are carried downstream along the characteristics. The flow upstream of the perturbations can be considered undisturbed.

The same pattern is seen for Taylor flows if the perturbations generated by irregularities of the wall inside the supersonic flow region die out within this region. If they do not, they reach the sonic line and enter the subsonic region. This disturbs the entire system, including the upstream flow. In this case, distortions which reach the wall also appear in the supersonic region. After reflection from the solid boundary, these distortions also contribute to the perturbations propagating away from the wall. In addition, perturbations arriving from the sonic line may themselves engender shock waves in the flow. However, this problem is outside the scope of the present study.

The system of equations obtained above can be reduced to a single first-order differential equation which is nonhomogeneous, nonlinear, and, in the general case, nonintegrable:

$$\frac{4}{3}\sqrt{\gamma+1}\frac{d}{dy}\left[(u_{11}+u_1)^{3/2} - u_1^{3/2}\right] = -\frac{2\nu}{3y}\sqrt{\gamma+1}\frac{d}{dy}\left[(u_{11}+u_1)^{3/2} - u_1^{3/2}\right] - Eu_{11} + C,$$
$$C = -\frac{\nu}{y}\left\{v_{11}(0) + \frac{2}{3}\sqrt{\gamma+1}\left[(u_{11}(0)+u_1(0))^{3/2} - u_1(0)^{3/2}\right]\right\}.$$

We rewrite this equation as follows:

$$\frac{du_{11}}{dy}=\frac{-\frac{2\nu}{3y}\sqrt{\gamma+1}\frac{d}{dy}\left[(u_{11}+u_1)^{3/2}-u_1^{3/2}\right]-Eu_{11}+C}{\sqrt{4(\gamma+1)(u_1+u_{11})}}-\frac{du_1}{dy}\frac{\sqrt{u_1+u_{11}}-\sqrt{u_1}}{\sqrt{u_1+u_{11}}}. \quad (1.13)$$

For normal gas dynamics ($\omega \to 0$, $E \to 0$) in the case of plane flow ($\nu = 0$), Eq. (1.13) has the form

$$\frac{du_{11}}{dy}=-\frac{du_1}{dy}\frac{\sqrt{u_1+u_{11}}-\sqrt{u_1}}{\sqrt{u_1+u_{11}}}.$$

It is evident from this that the perturbations die out if the undisturbed flow accelerates ($du_1/dy > 0$). Otherwise (if the flow is slowed ($du_1/dy < 0$) the perturbations grow. Relaxation occurring in the gas, permitting the excitation of vibrational degrees of freedom, leads the appearance of the term $-Eu_{11}$ in Eq. (1.13) and thus results in additional decay of the perturbations. In the case of axisymmetric flow ($\nu = 1$), divergence of the waves also weakens the distortions.

The characteristics represented by nonlinear equation (1.12) may intersect, with a shock wave being formed at the point of intersection. This point (the beginning of the shock wave) can be found from the equation $\partial y \backslash \partial y_0 = 0$. Let the solution of Eq. (1.13) have the form $u_{11} = u_{11}(y, y_0)$. Finding the derivative $\partial y/\partial y_0$ from (1.12), we represent the condition of intersection of the characteristics as

$$\frac{dy}{dy_0}=1-\frac{1}{2\sqrt{\gamma+1}}\int\frac{\partial u_{11}}{\partial y_0}\frac{1}{\left(\sqrt{u_1+u_{11}}\right)^3}dy=0,$$

It follows from this that the characteristics can intersect only when $\partial u_{11}/\partial y_0 > 0$, i.e., shock waves are obtained only from distortions containing sections on which the velocity increases in the reaction across the flow (compressive distortions).

2. Let us examine a system of equations of nonsteady quasi-unidimensional gas flow permitting excitation of vibrational degrees of freedom:

$$u_t+uu_x+\frac{1}{\rho}p_x=0,\ \ p_t+up_x+\rho a^2u_x=-\rho(\gamma-1)F-\rho ua^2\frac{1}{S}\frac{dS}{dx}, \quad (2.1)$$
$$T(s_t+us_x)=-F,\ \ e_{ht}+ue_{hx}=F,\ \ F=\omega\left(e_h^*-e_h\right).$$

Here, u is gas velocity; s is the entropy associated with the translational degrees of freedom of the gas molecules; S is the area of the cross section of the stream tube; and the subscripts x and t denote differentiation with respect to the corresponding coordinate.

The steady transonic flow whose stability we will study is described by a system of equations which follows from system (2.1) when we ignore the time dependence of the flow parameters in the latter:

$$\bar\rho\bar u\bar u_x+\bar p_x=0,\ \ \bar u\bar p_x+\bar\rho\bar a\bar u_x^2=-\bar\rho(\gamma-1)\bar F-\bar\rho\bar u\bar a^2\frac{1}{S}\frac{dS}{dx},\ \ \bar T\bar u\bar s_x=-\bar F,$$
$$\bar u\bar e_{hx}=\bar F,$$

From here, we easily obtain

$$\frac{d\bar u}{dx}=\frac{1}{\bar M^2-1}\left[\frac{(\gamma-1)}{\bar a^2}\bar F+\bar u\frac{1}{S}\frac{dS}{dx}\right]. \quad (2.2)$$

Let the velocity of the steady-state flow reach the speed of sound [$\bar u = \bar a$ (and $\bar M = 1$)] at the point $x = 0$ of the x axis. The existence of a continuous nonsteady flow with crossing of the sound barrier requires satisfaction of a condition which follows from (2.2):

$$\left|\frac{(\gamma-1)}{\bar a^3}\bar F=-\frac{1}{S}\frac{dS}{dx}\right|_{x=0}. \quad (2.3)$$

It follows from (2.2) that the release of energy as a result of vibrational excitation of molecules of the gas ($\bar{F} < 0$) accelerates the subsonic flow ($\bar{M}^2 < 1$) and slows the supersonic flow ($\bar{M}^2 > 1$). In a nonequilibrium gas with unexcited vibrational degrees of freedom ($\bar{F} > 0$), the subsonic flow is slowed due to the transfer of energy into internal degrees of freedom. The supersonic flow is accelerated in this case.

A change in the cross section of the stream tube (contraction or expansion) is analogous to the static effect of vibrational relaxation in the sense that contraction of the tube (for example) results in the same acceleration of the flow as relaxation of the vibrational degrees of freedom of the gas.

In a gas with frozen vibrational degrees of freedom, the sound barrier can be crossed only when $dS/dx = 0$ (minimum stream-tube cross section).

In accordance with (2.3), in a vibrationally excited gas ($\bar{F} < 0$) we should have $dS/dx > 0$ at $\bar{M} = 1$, and the crossing of the sound barrier takes place in the expanding part of the stream tube. In a nonequilibrium gas with unexcited vibrational degrees of freedom ($\bar{F} > 0$), it follows from (2.3) (with $\bar{M} = 1$) that we must have $dS/dx < 0$ – the crossing of the sound barrier occurs in the contracting part of the stream tube.

Another necessary condition for the existence of continuous transonic flow is stability flow in regard to small perturbations [8].

We will examine a small neighborhood of the point corresponding to the transition through the speed of sound $x = 0$ $[-\delta, \delta]$ (it is assumed that $\delta \ll 1$). We introduce a new space variable $x' = x/\delta$ and expand the flow parameters into series in the small quantity δ in the neighborhood of the point $x = 0$:

$$\begin{gathered} u = a_0 + \delta u_1(x', t) + \delta^2 u_2(x', t) + \ldots, \\ p = p_0 + \delta p_1(x', t) + \delta^2 p_2(x', t) + \ldots, \quad \rho = \rho_0 + \delta\rho_1(x', t) + \delta^2\rho_2(x', t) + \ldots, \\ s = s_0 + \delta s_1(x', t) + \delta^2 s_2(x', t) + \ldots, \quad e_k = e_{k0} + \delta e_{k1}(x', t) + \delta^2 e_{k2}(x', t) + \ldots. \end{gathered} \tag{2.4}$$

Here, the zero subscript denotes values of the parameters of the gas flow at the point $x = 0$. The subsequent terms correspond to deviations from the values at this point due to a steady change in the main flow and weak nonsteady distortions in this flow.

Inserting expansions (2.4) into system (2.1), in the zeroth approximation with respect to δ we have

$$\begin{gathered} \rho_0 a_0 u_{1x'} + p_{1x'} = 0, \qquad a_0 p_{1x'} + \gamma p_0 u_{1x'} = 0, \\ a_0 s_{1x'} = 0, \qquad a_0 e_{k1x'} = 0, \end{gathered}$$

from which

$$a_0 p_1 + \gamma p_0 u_1 = 0, \; s_1 = 0, \; e_{k1} = 0. \tag{2.5}$$

In the first approximation for δ, it follows from (2.1) that

$$\begin{gathered} p_{1t} + u_1 p_{1x'} + a_0 p_{2x'} + \gamma(p_1 u_{1x'} + p_0 u_{2x'}) = (\gamma - 1)\left[(\rho_1\omega_0 + \rho_0\omega_1)(e^*_{k0} - \right. \\ \left. - e_{k0}) + \rho_0\omega_0(e^*_{k1} - e_{k1})\right] - \left[\frac{a_0 p_1 + u_1 p_0}{a_0 p_0}\rho_0(\gamma - 1)\omega_0(e^*_{k0} - e_{k0}) + \right. \\ \left. + p_0 a_0\left(\frac{d^2 \ln S}{dx^2}\right)_0 x'\right], \\ \rho_0(u_{1t} + u_1 u_{1x'} + a_0 u_{2x'}) + \rho_1 a_0 u_{1x'} + p_{2x'} = 0, \\ s_{1t} + u_1 s_{1x'} + a_0 s_{2x'} = \frac{1}{T_0}\left(\frac{1}{T_0}\omega_0(e^*_{k0} - e_{k0})T_1 - \omega_1(e^*_{k0} - e_{k0}) + \omega_0(e^*_{k1} - e_{k1})\right), \\ e_{k1t} + u_1 e_{k1x'} + a_0 e_{k2x'} = \omega_1(e^*_{k0} - e_{k0}) + \omega_0(e^*_{k1} - e_{k1}). \end{gathered} \tag{2.6}$$

System (2.6) is degenerate relative to the unknowns p_2, u_2, s_2, and e_{k2} and has a solution only in the case of satisfaction of the compatibility condition for the system of equations

$$\begin{gathered} (\gamma - 1)\left[(\rho_1\omega_0 + \rho_0\omega_1)(e^*_{k0} - e_{k0}) + \rho_0\omega_0(e^*_{k1} - e_{k1})\right] - \\ - \left[\frac{a_0 p_1 + u_1 p_0}{a_0 p_0}\rho_0(\gamma - 1)\omega_0(e^*_{k0} - e_{k0}) + p_0 a_0\left(\frac{d^2 \ln S}{dx^2}\right)_0 x'\right] + p_{1t} - u_1 p_{1x'} - \gamma p_1 u_{1x'} + \rho_0 a_0(u_{1t} + u_1 u_{1x'}) + \rho_1 a_0^2 u_{1x'} = 0. \end{gathered}$$

TABLE 1

Gas	θ_k, K	T_k, K	ω, sec^{-1}	α, sec^{-1}	β, sec^{-2}	$\bar{x}$, m
N_2	3395	100	31,98	−0,0086	10^5	0,5
N_2	3395	300	31,98	−0,0033	10^5	0,5
N_2	3395	1000	31,98	15,02	10^5	0,506
N_2	3395	2000	31,98	97,11	10^5	0,5349
CO	3080	300	7199	−1,759	93 840	0,5
CO	3080	500	7199	159,4	92 550	0,5589
O_2	2239	300	$21\cdot10^5$	−4427	87 970	0,5

With allowance for (2.5), the compatibility condition reduces to the equation for $c=\frac{2\gamma+1}{\gamma+1}u_1$:

$$c_t + cc_{x'} = \alpha c + \beta x'. \tag{2.7}$$

Here

$$\alpha = -\frac{\gamma(\gamma-1)^2}{(\gamma+1)a_0^4}\omega_0\left[a_0^2(e_{k0}^* - e_{k0})\left(\frac{2\gamma}{\gamma-1}+\frac{k_2}{3T_0^{1/3}}\right)+\gamma e_{k0}^{*2}\exp\frac{\theta_k}{T_0}\right];$$

$$\beta = -\frac{(2\gamma+1)a_0^2}{(\gamma+1)^2 S_0}\left[\frac{1}{S_0}\left(\frac{dS}{dx}\right)_0^2 - \left(\frac{d^2S}{dx^2}\right)_0\right].$$

Equation (2.7) can be rewritten in the form of an independent system of ordinary differential equations:

$$\frac{dc}{dt} = \alpha c + \beta x', \quad \frac{dx'}{dt} = c. \tag{2.8}$$

The solutions of system (2.8) in the neighborhood of the singular point $c = x' = 0$ were studied qualitatively in [8]. It follows from the results that it is possible to classify flows of a vibrationally relaxing gas as follows when the sound barrier is crossed: stable transonic flow at $\alpha < 0$; unstable transonic flow at $\alpha > 0$, except for the case $\beta > 0$, $du_1/dx' > 0$. It should be noted that $\alpha > 0$ with sufficiently strong excitation of the vibrational degrees of freedom.

$$e_{k0} - e_{k0}^* > \frac{\gamma e_{k0}^{*2}\exp\frac{\theta_k}{T_0}}{a_0^2\left(\frac{2\gamma}{\gamma-1}+\frac{k_2}{3T_0^{1/3}}\right)}. \tag{2.9}$$

Let us examine specific conditions for flows of a vibrationally relaxing gas. If we study a flow of an equilibrium gas, we find that $e_{k0}^* = e_{k0}$. In the minimum cross section of the stream tube $(dS/dx)_0 = 0$ and $(d^2S/dx^2)_0 > 0$, so that $\alpha < 0$ and $\beta > 0$. In this case, flows crossing the sound barier from either region (from subsonic to supersonic or from supersonic to subsonic) are stable.

Sufficiently strong excitation of the vibrational degrees of freedom (see (2.9)) causes α to be positive. Here, $e_{k0}^* < e_k^0$ and $(dS/dx)_0 > 0$. If in this case $\beta < 0$, (i.e.,) $(d^2S/dx^2)_0 < (dS/dx)_0^2/S_0$ then the transonic flow becomes unstable (this includes flows undergoing a transition of the subsonic-supersonic type). Thus, a flow changing from the subsonic to the supersonic regime is unstable in a vibrational excited gas flowing through a channel with a cross section that decreases at a constant or increasing rate. If $\beta > 0$ (when the stream tube expands very rapidly), then transonic flow with a subsonic-supersonic transition is stable and flow with a supersonic-subsonic transition is unstable.

If the gas is nonequilibrium in the sense that $e_{k0}U^* > e_{ko}$, then $\alpha < 0$. However, in this case $(dS/dx)_0 < 0$, and the cases $\beta < 0$ (constant or increasing contraction of the stream tubes) and $\beta > 0$ (decreasing contraction of the stream tubes) are possible. All possible transonic flows are stable in both the first and second cases.

Let us calculate the coefficients α and β for specific gases on the basis of the data in [14]. Let the cross-sectional area of the stream tube be determined from the formula

$$S = 1 - \bar{x}(1 - \bar{x}),\ 0 \leqslant \bar{x} \leqslant 1.$$

The following values are taken for pressure and temperature: p = 101,320 Pa, T = 300 K. In accordance with the given state parameters of the gas, we determine the critical point and calculate α and β at this point. The results of the calculations, presented in Table 1, show that vibrational excitation of molecules of nitrogen and carbon monoxide changes stable transonic flows of these gases into unstable flows. Transonic flows of molecular oxygen are stable, since vibrational excitation of the molecules of this gas rapidly relaxes to equilibrium.

The results obtained here can be used in problems involving the transonic flow of a gas through a nozzle and problems concerning flow about a body in the case when a local supersonic region is formed.

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